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Parameter Estimation of Linear MIMO Systems Using Sequential Relay Feedback Test

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The interactions between inputs and outputs in the multi input multi output (MIMO) systems yield responses from closed-loop off-diagonal elements. To characterize these responses through modeling, a systematic approach is followed to derive analytical expressions for relay feedback responses on multivariable systems. Time domain analytical expressions are helpful in deriving boundary conditions to estimate the parameters of MIMO systems. Unknown system parameters are identified using limit cycle data of off-diagonal closed-loop relay response curves obtained from single relay feedback test on these MIMO systems. Developed models are found to be in close agreement with real time system, and the parametric identification procedure is also efficient. © 2014 American Institute of Chemical Engineers *AIChE J*, 60: 1672–1681, 2014
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Introduction

The majority of industrial plants contain multivariable control loops, that is, there are many output variables to be controlled, and more than one input variable is coupled with the outputs. Most of the closed-loops use proportional-integral-derivative (PID) controllers due to their ease in implementation and maintenance. Proper tuning (model based) of PID loops needs proper identification of process models. Internal model control-PID tuning needs values of process model parameters. Relay feedback is one promising tool for identification of process models in real time. Relay feedback is important as it is used in auto tuning of processes. The relay feedback test has been first proposed by Astrom and Hagglund[1] for process control practitioners as a closed-loop tool for system identification and control by introducing periodic step functions as input to the unknown process and collecting outputs (that lags behind input by π radians) from which limit cycle data (Ku and ωu) are found to approximate the process model parameters. Many researchers (Luyben,[2] Yu,[3] Majhi and Atherton,[4] Vrancic et al.,[5] Leva et al.[6]) have worked on auto tuning of PID controller for low order systems and have reported of getting more information and better performance of closed-loop system. But these processes are single input single output in nature where there is no interaction between input and output. As chemical processes are dominated in time delay and most of them are multivariable in nature, Shen and Yu[7] presented relay feedback-based parameter estimation methods for MIMO systems. Luyben[2] discussed a very simple technique that needs one additional parameter (other than limit cycle data), namely, shape factor, to identify process transfer functions using a single relay feedback test for stable and unstable first order plus dead time (FOPDT) systems. The shapes of relay response curves primarily give an idea of the system category and its order. Thyagarajan and Yu[8] categorized process models by observing the shapes of relay feedback response (generated from mostly FOPDT processes with different D/τ ratio and higher order systems) and identified the transfer function models. Panda and Yu[9] identified the parameters of second order plus dead time processes with the help of analytical expressions of relay response along with the boundary conditions. Panda et al.[10] estimated the parameters of several integrating processes using land mark points information on relay responses. Lee et al.[11] used two area of relay feedback response to compute the parametric models. Liu and Gao[12,13] estimated parameters for low order single input, single output systems. Gu et al.[14] explained methods of estimating model parameters for integrator plus dead time systems. Lee et al.[15] used area method to estimate model parameters of FOPDT systems. Tsang et al.[16] presented auto tuning methodologies for integrating systems. Most of these aforementioned methods are limited to single input single output (SISO) techniques, and none of them use information from off-diagonal elements for identifying process parameters. As the load disturbances drive the system away from its desired behavior, hence the load responses are defined as response from closed-loop off-diagonal elements. Relay response for set point tracking (or direct loop or diagonal closed transfer function) is termed as desired relay response, and the relay response in which load disturbances drive the system away from its desired behavior (or indirect loop or off-diagonal closed loop transfer function) is called as relay response from closed-loop off-diagonal elements. Models of response from closed-loop off-diagonal elements are used for analyzing interactions and subsequent justification/selection of input–output pairs for control.

The multivariable auto tuning design methods ensure that the closed-loop process works near critical frequency regions. Many available decentralized control methods consider desired response (for example in case of 2-by-2 system: output (y_1) and input (u_1) for system identification and analysis and they do not discuss about methods to reduce interactions nor do they give any exact analytical expressions for

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the relay responses that may help in analyzing the interaction behavior between input/output and may provide information regarding closed-loop parameters (PID using model based tuning rules) of the MIMO system. Moreover, model parameters and information on interactions can be obtained from time-domain mathematical model of relay responses for MIMO systems. As the off-diagonal closed loop transfer functions contain information on interactions, it is needed to analyze the control system based on their time domain characteristics. If interactions can be reduced, that is, effects of off-diagonal elements can be made to almost negligible, then response from closed-loop off-diagonal elements can be minimized and closed-loop performance can be improved. Sujatha and Panda[17] recently presented novel techniques to reduce interaction and thereby minimizing optimal inputs to multivariable controllers. Thus, finding out the root cause of interactions and building up a mathematical model for it is behind the novel objective of this work. In the second stage, model parameters from unknown MIMO process are identified using relay responses from closed-loop off-diagonal element.

We consider a multi-input multioutput plant with m inputs and l outputs. Thus, a basic transfer function model is $y(s) = G(s)u(s)$, where, $y(s)$ is $l \times 1$ vector, $u(s)$ is $m \times 1$ vector, and $G(s)$ is $l \times m$ transfer function matrix. If we make a change in first input u_1, then this will generally affect all the outputs $y_1, \ldots y_l$, that is, there is interaction between inputs and outputs. In this article, 2-by-2 and 3-by-3 multi-input and multioutput systems with square matrix structures and whose individual/component transfer functions are of FOPDT type are considered. The need for multivariable control in case of a continuous distillation column is to reduce interactions between top products and feed, bottom draw and feed. In fact, each unit needs control of at least two variables, product rate and quality, requiring two control loops.

Auto tuning offers an efficient and recent tool to estimate and tune controller parameters. In this article, relay feedback approach is used to identify model parameters and controller parameters of 2-by-2 and 3-by-3 MIMO systems. Relay responses from closed-loop off-diagonal elements are obtained from theoretical equations and validated against the simulated relay response, and parameter estimation algorithms are formulated using land mark points. The objective of the work is as follows: analysis of interaction will be helpful if we model the relay response from closed-loop off-diagonal elements, and we may be able to identify process as well as controller parameters from the response.

This article is organized as follows: introduction to relay feedback test on MIMO process is detailed in section "modeling" followed by derivation of analytical expressions of relay responses (from off-diagonal decentralized closed-loop transfer function) for typical 2-by-2 and 3-by-3 MIMO systems in section "identification." In section "validation," responses of closed-loop off-diagonal elements obtained from simulating relay-analytical expressions are validated against that obtained from experimental (real time 2-by-2 liquid level control coupled system and simulink models) response. Parameter estimation from experimental response of 2-by-2 as well as 3-by-3 MIMO systems is presented in this section, and concluding remarks are drawn in the final section.

Time Domain Mathematical Modeling of MIMO Processes

Modeling of 2-by-2 MIMO system

Based on the concept of sequential auto tuning (Shen and Yu[7]) method each controller is designed in sequence. Let us consider a 2-by-2 MIMO system with a known pairing (y_1-u_1) and (y_2-u_2) under decentralized proportional-integral control. Initially, an ideal/biased relay is placed between y_1 and u_1 while Loop 2 is on manual. Following the relay-feedback test, a controller can be designed from the ultimate gain and ultimate frequency. Multiloop PI controller is used here. The next step is to perform relay-feedback test between y_2 and u_2 while Loop 1 is on automatic.

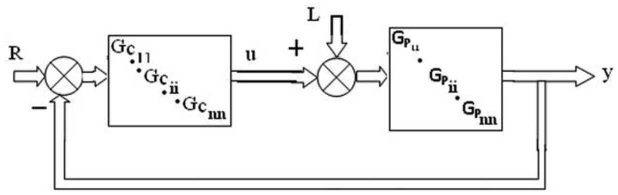

Figure 1. Block diagram of n-by-n multivariable systems with decentralized PI controller.

Consider the n-by-n MIMO system whose block diagram is given in Figure 1. Let us consider squared 2-by-2 and 3-by-3 MIMO systems whose structures are given in Table 1.

A controller can also be designed for Loop 2 following the relay-feedback test. Once the controller on the Loop 2 is put on automatic, another relay-feedback experiment is performed between y_1 and u_1. Generally, a new set of tuning constants are found for the controller in Loop 1. This procedure is repeated until the controller parameters converge. Typically, the controller parameters converge in 3–4 relay-feedback tests for 2-by-2 systems. After developing the theoretical transfer function models for off-diagonal closed-loop elements, the time response can be obtained by simulation. The interactive closed loop transfer functions for 2-by-2 system can be written as

$$g_{p21,cl}=\frac{g_{p21}}{1+G_{C2}g_{p22}} \text{ where } g_{p21}=\frac{k_{21}e^{-D_{21}s}}{\tau_{21}s+1}, \quad g_{p22}=\frac{k_{22}e^{-D_{22}s}}{\tau_{22}s+1}, \text{ and } G_{C2}=K_{C2}\left(1+\frac{1}{\tau_{I2}s}\right) \tag{1}$$

The effective closed loop relation between y_2 and u_1 for 3-by-3 MIMO system is

$$g_{p21,cl}=\frac{g_{p21}}{1+G_{C2}g_{p22}}-\frac{g_{p23}g_{p31}}{G_{C3}g_{p33}\left(1+G_{C3}g_{p33}\right)} \tag{2}$$

where $g_{p21}=\frac{k_{21}e^{-D_{21}s}}{\tau_{21}s+1}$; $g_{p22}=\frac{k_{22}e^{-D_{22}s}}{\tau_{22}s+1}$; $g_{p23}=\frac{k_{23}e^{-D_{23}s}}{\tau_{23}s+1}$; $g_{p31}=\frac{k_{31}e^{-D_{31}s}}{\tau_{31}s+1}$; $g_{p33}=\frac{k_{33}e^{-D_{33}s}}{\tau_{33}s+1}$; $G_{C2}=K_{C2}\left(1+\frac{1}{\tau_{I2}s}\right)$; $G_{C3}=K_{C3}\left(1+\frac{1}{\tau_{I3}s}\right)$

The ideal relay response is assumed to be formed by n-number of infinite-small step changes. Let $\mu_+ = \mu_0 + \mu$, $\mu_- = \mu_0 - \mu$. The process input in the relay feedback test consists of series of step changes with down amplitude, μ_- and up amplitude, μ_+. Here, μ_-, μ_+ denote fluctuations (positive for more and negative for less) of relay input from nominal value μ_0. At the first interval, (after synchronizing input with output by time shift) the response for Eq. 1 can be described as

$$y_1=k_{21}\mu_0\left[1-e^{-t/\tau_{21}}\right] - \frac{k_{22}K_{C2}}{\tau_{I2}}\mu_-\left[t-(\tau_{I2}+\tau_{22})(1-e^{-t/\tau_{22}})\right]y_1(t-1) \tag{3}$$

At the second instant, the time is reset to zero at the initial point

$$y_2 = k_{21}\mu_0\left\{[1-2]-e^{-\frac{t}{\tau_{21}}}\left[e^{-\frac{D_{21}}{\tau_{21}}}-2\right]\right\}- \\ -\frac{k_{22}K_{C2}}{\tau_{I2}}\mu_-\left\{t[1-2]+\left((\tau_{I2}-\tau_{22})\left(1-e^{-\frac{t}{\tau_{22}}}\right)\left[e^{-\frac{D_{22}}{\tau_{22}}}-2\right]\right)\right\}y_2(t-1) \tag{4}$$

As time tends to infinity, the response becomes stabilized and it can be described as $y_n = y_1+y_2+y_3+$.........

Thus, the generalized analytical expressions for 2-by-2 MIMO system is given by

$$y_n = \left(k_{21}\mu_+ - 2k_{21}\mu_0 e^{-\frac{t}{\tau_{21}}}\left(\frac{1-e^{-\frac{p-}{\tau_{21}}}}{1+e^{-\frac{p_u}{2\tau_{21}}}}\right)\right) \\ -\left(\frac{k_{22}K_{C2}}{\tau_{I2}}(\mu_- t_1+\mu t)-\frac{k_{22}K_{C2}}{\tau_{I2}}(\tau_{I2}+\tau_{22}) \right. \\ \left.\left[\mu_+ - 2\mu e^{-\frac{t}{\tau_{22}}}\left(\frac{1-e^{-\frac{p-}{\tau_{22}}}}{1+e^{-\frac{p_u}{2\tau_{22}}}}\right)\right]\right)y_n(t-1) \tag{5}$$

The term $y_n(t-1)$ in Eq. 5 is predicted using one step ahead prediction algorithm. After two or three full oscillations, the relay response is found to be stabilized.

Modeling of 3-by-3 MIMO system

For 3-by-3 MIMO systems, the relay responses (Figure 2a) obtained from interactive transfer function (Eq. 2) are modeled as follows. The relay response is assumed to be formed by *n*-number of infinite-small step changes. At the first interval, the response can be described as

$$y_1 = \left(k_{21}\left[1-e^{-t/\tau_{21}}\right]-\frac{k_{22}K_{C2}}{\tau_{I2}}\left[1+(\tau_{I2}-\tau_{22})e^{-t/\tau_{22}}\right]\right)y_1(t-1) \\ -\left(\frac{k_{23}k_{31}\tau_{I3}}{K_{C3}k_{33}}\left[1-\left(a_1e^{-t/\tau_{13}}+a_2e^{-t/\tau_{23}}+a_3e^{-t/\tau_{33}}\right)\right] \right. \\ \left. -\frac{k_{33}K_{C3}}{\tau_{I3}}\left[1+(\tau_{I3}-\tau_{33})e^{-}t/\tau_{33}\right]\right)y_1(t-1) \tag{6}$$

$$\text{where } a_1=\frac{\tau_{I3}(\tau_{I3}-\tau_{33})}{(\tau_{I3}-\tau_{23})(\tau_{I3}-\tau_{31})};\ a_2=\frac{\tau_{23}(\tau_{23}-\tau_{33})}{(\tau_{23}-\tau_{I3})(\tau_{23}-\tau_{31})}; \\ a_3=\frac{\tau_{31}(\tau_{31}-\tau_{33})}{(\tau_{31}-\tau_{I3})(\tau_{31}-\tau_{23})} \tag{7}$$

Let $D = D_{23} + D_{31} - D_{33}$, at the second instant, the relay output can be given by

$$y_2 = \left(k_{21}\left\{[1-2]-e^{-t/\tau_{21}}\left[e^{-D_{21}/\tau_{21}}-2\right]\right\}-\frac{k_{22}K_{C2}}{\tau_{I2}}\left\{[1-2]+\left((\tau_{I2}-\tau_{22})e^{-t/\tau_{22}}\left[e^{-D_{22}/\tau_{22}}-2\right]\right)\right\}\right)y_2(t-1) \\ -\left(\begin{array}{l}\frac{k_{23}k_{31}\tau_{I3}}{K_{C3}k_{33}}\left[\left\{1-\left(a_1e^{-t/\tau_{13}}\left[^{-D/\tau_{13}}-2\right]+a_2e^{-t/\tau_{23}}\left[^{-D/\tau_{23}}-2\right]+a_3e^{-t/\tau_{31}}\left[^{-D/\tau_{31}}-2\right]\right)\right\}\right] \\ -\frac{k_{33}K_{C3}}{\tau_{I3}}\left\{[1-2]+\left((\tau_{I3}-\tau_{33})e^{-t/\tau_{33}}\left[e^{-D/\tau_{33}}-2\right]\right)\right\}\end{array}\right)y_2(t-1) \tag{8}$$

As time tends to infinity, the response becomes stabilized in terms of amplitude and period of oscillation. Thus, the overall response can be described as $y_n = y_1 + y_2 + y_3 + \ldots\ldots\ldots$

The generalized analytical expressions for 3-by-3 system is given by

$$y_n = \left(k_{21}\left(1-e^{-t/\tau_{21}}\left(\frac{2}{1+e^{-p_u/2\tau_{21}}}\right)\right)-\frac{k_{22}K_{C2}}{\tau_{I2}}\left(1+(\tau_{I2}-\tau_{22})e^{-t/\tau_{22}}\left(\frac{2}{1+e^{-p_u/2\tau_{22}}}\right)\right)\right)y_n(t-1) \\ -\left(\begin{array}{l}\left[1-\left(a_1e^{-t/\tau_{13}}\left(\frac{2}{1+e^{-p_u/2\tau_{13}}}\right)+a_2e^{-t/\tau_{23}}\left(\frac{2}{1+e^{-p_u/2\tau_{23}}}\right)+a_3e^{-t/\tau_{33}}\left(\frac{2}{1+e^{-p_u/2\tau_{33}}}\right)\right)\right] \\ -\frac{k_{33}K_{C3}}{\tau_{I3}}\left[1+(\tau_{I3}-\tau_{33})e^{-t/\tau_{33}}\left(\frac{2}{1+e^{-p_u/2\tau_{33}}}\right)\right]\end{array}\right)y_n(t-1) \tag{9}$$

The term $y_n(t-1)$in Eq. 9 is one step ahead prediction of $y_n(t)$.

Identification of Parameters with Relay Feedback

Identification of parameters of 2-by-2 MIMO system

Relay response for set point tracking (or direct loop or diagonal closed transfer function) is termed as desired relay response, and the relay response in which load disturbances drive the system away from its desired behavior (or indirect loop or off-diagonal closed loop transfer function) is called as relay response from closed-loop off-diagonal elements. For 2-by-2 MIMO system, interactive off-diagonal closed-loop transfer function is having six unknown parameters to be estimated in which D_{21} can be measured from stable part of relay response from closed-loop off-diagonal elements, D_{22} can be obtained directly from initial part of desired relay response, and remaining four unknown parameters are k_{21}, t_{21}, k_{22}, t_{22}. Basically, when one identifies an unknown process in closed-loop, the controller parameters, K_C and τ_I, are known. After observing and analyzing the off-diagonal closed-loop response, the boundary conditions can be formulated using land mark points. For the monotonic (responses have sharp cornered edges or peaks), the boundary conditions becomes as

Table 1. Model Structures and Off-Diagonal Transfer Functions of Multivariable Systems

2-by-2 System	3-by-3 System
$g_p(s)=\begin{pmatrix} g_{p11} & g_{p12} \\ g_{p21} & g_{p22} \end{pmatrix}$	$g_p(s)=\begin{pmatrix} g_{p11} & g_{p12} & g_{p13} \\ g_{p21} & g_{p22} & g_{p23} \\ g_{p31} & g_{p32} & g_{p33} \end{pmatrix} G_C(s)=\begin{pmatrix} G_{C1} & 0 & 0 \\ 0 & G_{C2} & 0 \\ 0 & 0 & G_{C3} \end{pmatrix}$
$G_C(s)=\begin{pmatrix} G_{C1} & 0 \\ 0 & G_{C2} \end{pmatrix}$	$u(s)=[u_1(s) \quad u_2(s) \quad u_3(s)]^T y(s)=[y_1(s) \quad y_2(s) \quad y_3(s)]^T$
$u(s)=[u_1(s) \quad u_2(s)]^T$	
$y(s)=[y_1(s) \quad y_2(s)]^T$	
$g_{p21,cl}=\dfrac{g_{p21}}{1+G_{C2}g_{p22}}$	$g_{p21,cl}=\dfrac{g_{p21}}{1+G_{C2}g_{p22}}-\dfrac{g_{p23}g_{p31}}{G_{C3}g_{p33}(1+G_{C3}g_{p33})}$

$$(y_m)_{t=0}=-a \tag{10}$$

$$(y_m)_{t=\frac{p_u}{2}}=+a \tag{11}$$

$$(y_m)_{t=\frac{p_u}{2}-D_{21}}=0 \tag{12}$$

$$(y_m)_{t=\frac{p_u}{2}-D^*}=0 \tag{13}$$

Thus, following steps are followed to estimate parameters of $g_{p21,cl}$ of 2-by-2 MIMO system using lower trenches of relay curves:

(Step 1): Find out D_{21} from stable part of off-diagonal closed-loop relay responses of 2-by-2 MIMO system or by applying the boundary condition: $(y_m)_{t=\frac{p_u}{2}-D_{21}}=0$ and solving for unknown parameter say D_{21}.

(Step 2): Estimate D_{22} from stable or initial part of desired relay responses of 2-by-2 MIMO system.

(Step 3): Solve Eqs. 10–14 simultaneously to find unknown parameters k_{21}, t_{21}, k_{22}, t_{22}. Using following boundary conditions

$$(y_m)_{t=0}=-a$$

$$(y_m)_{t=\frac{p_u}{2}}=+a$$

$$(y_m)_{t=\frac{p_u}{2}-D_{21}}=0$$

$$(y_m)_{t=\frac{p_u}{2}-D^*}=0$$

After identification, the results of estimating 2-by-2 system parameters are shown in Table 2. True and estimated parameters of Wood and Berry (WB) column are given in Table 3. Modeling error analysis is carried out in a latter section in order to justify the achievable accuracy of derived model to the real one.

Identification of parameters of 3-by-3 MIMO system

From the monotonic responses (Figure 3), t_{peak} and apparent dead time D^* can be obtained. t_{peak} is the time taken by the model to reach peak value of amplitude or response for a particular trench. D^* is called apparent dead time and is defined as time taken by the experimental response to reach peak of relay response. For 3-by-3 MIMO system, disturbance dynamics are captured by five process-transfer functions and two controllers. Each process is found to have FOPDT model structures. Hence, for five processes apparent dead times, the time taken by the response to reach its peak, are obtained directly from the output responses. D_{22}, D_{33} are obtained directly from the desired responses y_2, y_3 of Loop 2 and Loop 3, respectively. D_{21} is obtained directly from the stable part of off-diagonal closed-loop output response. Let Loop 1 be in manual, and Loop 2 and Loop 3 are in auto mode. D_{23} can be obtained from initial part of response y_2 of Loop 2 when Loop 3, ($y_{3\text{sp}}$) is excited. Similarly, D_{31} can be obtained from y_3 of Loop 3 when Loop 1, $y_{1\text{sp}}$ is excited.

Out of 19 parameters of disturbance dynamics, five apparent dead times are obtained directly from the response, four parameters of two controllers are known and hence remaining 10 parameters are estimated based on 10 equations which are formulated as follows: four equations are synthesized based on landmark points, four equations are formulated based on area of relay oscillation, two equations are based on moment methods.

Landmark points

Landmark points on the output response are starting point ($t=0$) and ending point ($t=P_u/2$), t_{peak} and D^* can be found easily as shown in Figure 3. The boundary conditions can be found from these landmark points as follows:

The boundary conditions are

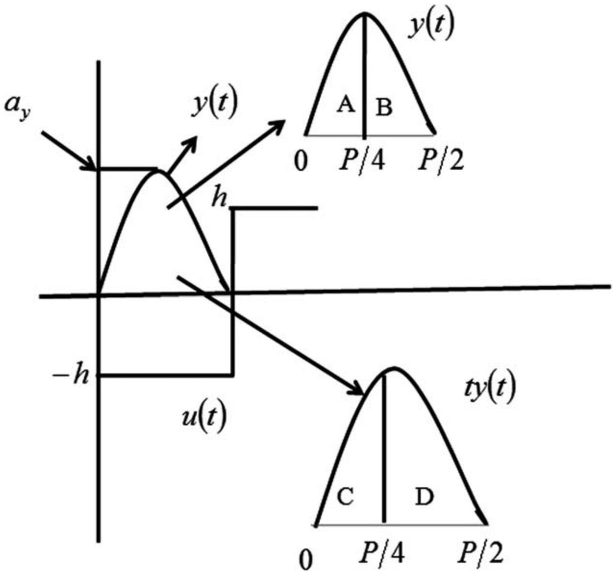

Figure 2. Area between relay feedback response and base line.

Table 2. Estimation of Parameters of Single Input-Output Cases

	Actual or True Process ($G_p(s)$)	Model or Estimated Process ($G_m(s)$)	Relay Response Properties	Multiplicative Error $e_m=\int_{\omega_u/\sqrt{10}}^{\omega_u*10}\frac{\text{abs}\left(G_p(j\omega)-G_m(j\omega)\right)}{G_p(j\omega)}d\omega$
Case 1	$\frac{0.2e^{-5s}}{(s+1)}$	$\frac{0.2000e^{-5s}}{(0.9997s+1)}$	$a=0.163$; $P_u=3.6796$	0.00000028009
Case 2	$\frac{1}{(s+1)^3}$	$\frac{0.163e^{-0.02s}}{(0.9997s+1)}$	$a=0.1257$; $P_u=14.8991$	0.0042

$$(y_m)_{t=0}=-a \tag{14}$$

$$(y_m)_{t=\frac{p_u}{2}}=+a \tag{15}$$

$$(y_m)_{t=\frac{p_u}{2}-D_{21}}=0 \tag{16}$$

$$(y_m)_{t=\frac{p_u}{2}-D^*}=0 \tag{17}$$

Area of relay oscillation

First, we consider the following two areas as shown in Figure 2

$$A=\int_{0}^{p_{u/4}} y(t)dt \tag{18}$$

$$B=\int_{p_{u/4}}^{p_{u/4}} y(t)dt \tag{19}$$

These areas can describe the integral

$$q_1=\int_0^{p_u} q\left(t-\frac{p_u}{4}\right)y(t)dt=2\left(\int_0^{p_{u/4}} y(t)dt+\int_{p_{u/4}}^{p_{u/2}} y(t)dt\right) \tag{20}$$

$$q_1=2(A+B)$$

$$q_2=\int_0^{p_u} q\left(t-\frac{p_u}{4}\right)y(t)dt=2\left(\int_0^{p_{u/4}} y(t)dt-\int_{p_{u/4}}^{p_{u/2}} y(t)dt\right) \tag{21}$$

$$q_2=2(A-B)$$

In addition to the above relations regarding areas, two more areas as shown in Figure 2 are considered for better estimates

$$C=\int_0^{p_{u/4}} ty(t)dt \tag{22}$$

$$D=\int_{p_{u/4}}^{p_{u/2}} ty(t)dt \tag{23}$$

Table 3. Comparison of True and Estimated Parameters of Interactive Closed-Loop Transfer Function of WB Column

WB Process	True Value	Estimated Value
k_{21}	6.6	6.5
t_{21}	10.9	11
D_{21}	7	7
k_{22}	−19.4	−19.408
t_{22}	14.4	14.388
D_{22}	3	3

Thus, we can formulate some more relations using the above two expressions for areas

$$q_3=\int_0^{p_u} r(t)y(t)dt=2\left(\int_0^{p_{u/4}} ty(t)dt+\int_{p_{u/4}}^{p_{u/2}}\left(\frac{p_u}{4}-t\right)y(t)dt\right) \tag{24}$$

$$q_3=2(C-D)+P_uB$$

$$q_4=\int_0^{p_u} r(t)y(t)dt=2\left(\int_0^{p_{u/4}}\left(\frac{p_u}{4}-t\right)y(t)dt+\int_{p_{u/4}}^{p_{u/2}}\left(\frac{p_u}{4}-t\right)y(t)dt\right)$$

$$q_4=-2(C+D)+\frac{P_u}{2}(A+B) \tag{25}$$

Moment method

Moment of a function can be used as a measure to estimate certain parameters of a function. We define the nth moment of a transfer function $g(t)$ about the origin as

$$m_n=\int_0^{\infty} t^n g(t)dt$$

Zeroth moment is given by Eq. 26

$$m_0=\int_0^{\infty} g(t)dt=\lim_{s\to 0} G(s) \tag{26}$$

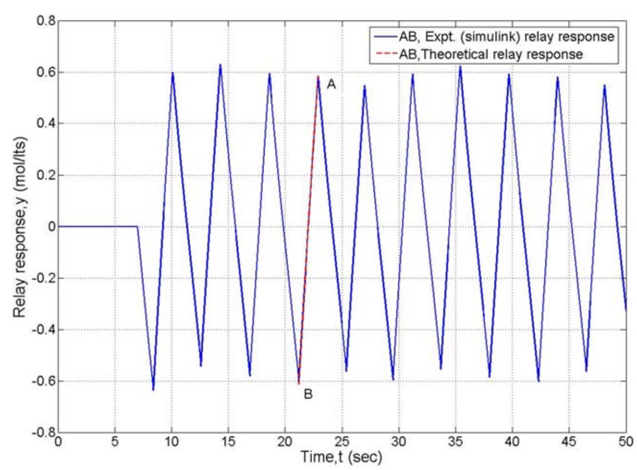

Figure 3. Validation of analytical expression (dashed) of 2 × 2 interactive cross transfer function with experimental (simulated) biased relay response (solid) for WB column.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Table 4. True and Estimated Process Model Parameters of OR Process

S. No.	Parameters	True Value	Estimated Value (Without Noise)	Estimated Value (With Noise)
1	k_{21}	1.11	1.11	1.09
2	t_{21}	3.25	3.249	3.245
3	D_{21}	6.6	6.6	6.6
4	k_{22}	−2.36	−2.356	−2.35
5	τ_{22}	5	5.003	4.98
6	D_{22}	3	3	3
7	k_{23}	−0.01	−0.01	−0.009
8	t_{23}	7.09	7.09	7.078
9	D_{23}	1.2	1.2	1.2
10	k_{31}	−34.68	−34.68	−34.672
11	t_{31}	8.15	8	8.07
12	D_{31}	9.2	9.2	9.2
13	k_{33}	0.68121	0.68	0.678
14	t_{33}	5.06	5	5
15	D_{33}	1	1	1

First moment is given by Eq. 27

$$m_1=\int_0^\infty tg(t)dt=\lim_{s\to 0}\frac{dG(s)}{ds} \tag{27}$$

Using above mentioned conditions (Eqs. 14–17; Eqs. 20 and 21; Eqs. 24 and 25; and Eqs. 26 and 27) on output response from off-diagonal closed-loop element, 10 equations can be synthesized. These 10 equations involving the unknown parameters are formulated using the boundary conditions and these equations are solved simultaneously using "fsolve" command available in MATLAB toolbox.

Thus, following steps are followed to estimate parameters of 3-by-3 MIMO system using lower trenches of relay curves:

(Step 1): Find out D_{21} from stable part of off-diagonal closed-loop relay response of 3-by-3 MIMO system.

(Step 2): Estimate D_{23} and D_{31} from stable or initial part of desirable and off-diagonal closed-loop relay responses of 3-by-3 MIMO system.

(Step 3): Solve Eqs. 14–17; Eqs. 20 and 21; Eqs. 24 and 25; and Eqs. 26 and 27 simultaneously to find unknown parameters k_{21}, t_{21}, k_{22}, t_{22}, k_{23}, t_{23}, k_{31}, t_{31}, k_{33}, t_{33}.

Similarly, the same procedure is carried out to estimate the parameters of other off-diagonal closed-loop transfer functions by simply changing the suffixes.

After identification, the estimated parameters of 3-by-3 system are shown as in Table 4. The efficiency of the present method of estimation in identifying process model (Ogunnaike–Ray (OR) process) parameters can be judged by evaluating multiplicative error of the identified model with reference to the real process around its ultimate frequency. This has been reported at a latter section.

Validation of Model Structures for SISO/MIMO System

SISO cases

We consider two examples, one FOPDT example and another $G_p(s)=1/(s+1)$.[3] We use stabilized response and estimate apparent dead time and transfer function as FOPDT from stabilized response.

Apparent dead times are the time taken by the response to reach its peak. Estimation of the dead time has been done (instead of taking from initial response we use land mark points and boundary conditions on relay equation) using boundary conditions applied on analytical equations of relay response. For finding out the time delay (D_p), we apply the boundary condition that at $t=0.5P_u-D_p$, $y_n=0$. The relay response model for an FOPDT system can be given as $y_n=K_P\left\{1-e^{-t/\tau}\left(\frac{2}{1+e^{-0.5P_u/\tau}}\right)\right\}$. After applying the boundary conditions, we get from the above equation, $D_P=0.5P_u+\tau\ln\left(\frac{1+e^{-0.5P_u/\tau}}{2}\right)$. The results are shown in Table 2. In Case 1, apparent dead time is 5, whereas, in Case 2, apparent dead time becomes 0.02.

Validation of 2-by-2 MIMO process

WB Distillation Column. The transfer function of WB distillation column is given by

$$\begin{bmatrix} x_D \\ x_B \end{bmatrix}=\begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}\begin{bmatrix} L \\ V \end{bmatrix}+\begin{bmatrix} \dfrac{3.8e^{-8.1s}}{14.9s+1} \\ \dfrac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix}[F] \tag{28}$$

The compositions of top (x_D) and bottom (x_B) products expressed in wt % of methanol are controlled variables. The reflux (L) and the reboiler steam flow rates (V) are the manipulated inputs, expressed in lb/min, and time constants are in min. Feed flow rate (F) is disturbance.

Sequential relay feedback tests are carried out on the WB column. Figure 3 shows the validation of derived mathematical models where theoretical response matches exactly with the simulated one.

For WB process, D_{21} and D_{22} are found to be 7 and 3, comparison of true value with estimated value is listed in Table 3 and validation is shown in Figure 3.

Coupled Tanks System. The coupled tanks (four tanks connected to each other in a network) setup is a model of a liquid storage system in process industries as shown in Figure 4a. Often tanks are coupled through connecting pipes. These storage facilities contain fluids where the reactant level and flow are to be controlled. Water is chosen as the fluid. The experiment setup of coupled tanks is designed so that the system can be configured. The setup has four translucent tanks each with a pressure sensor to measure the water levels. The couplings between the tanks can be modified by the use of seven manual valves to change the dynamics of the system imposing the use of different controllers. Water is delivered to the tanks by two independently controlled, submersed pumps. Step disturbances on the flow rate are provided by four manual valves. Drain flow rates can be

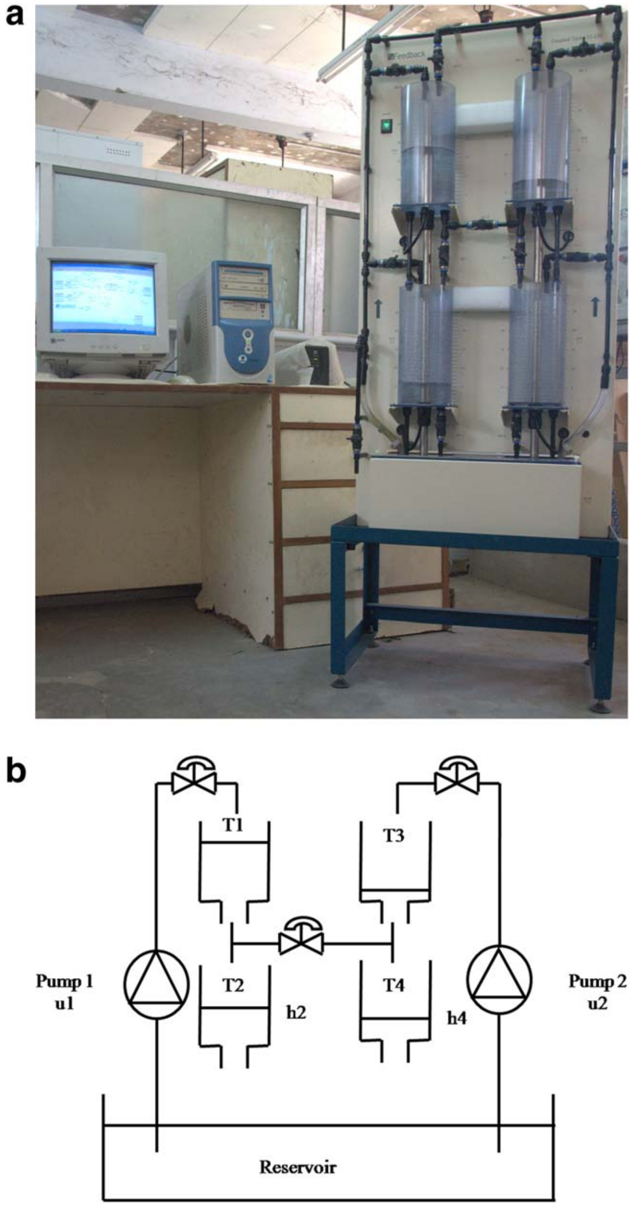

Figure 4. (a) Experimental setup for coupled tanks system (liquid level); (b) schematic diagram of coupled tanks experimental setup.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

modified using orifice caps that are easy to change. The coupled tanks system is interfaced with computer through MATLAB/SIMULINK and an Advantech PCI 1711 data acquisition interface card.

The schematic of the coupled tanks experimental system as shown in Figure 4b has been used to control the levels in the lower two tanks with two pumps. The process inputs are u_1 and u_2 (input voltages to excite the pumps) and the outputs are h_4 and h_2 (heights from level measurement devices). The design parameters of the coupled tanks experimental set up are cross-sectional area of each tank, $A = 0.01389\ \text{m}^2$; outlet (exit pipe) area of tank, $a = 50.2658 \times 10^{-6}\ \text{m}^2$ and gravitational constant, $g = 9.81\ \text{m/s}^2$. A step disturbance on u_1 is given and changes in h_4 and h_2 are recorded. Similar step responses in levels for step disturbance on u_2 are also collected. These responses are shown in Figure 5.

Formulation of process model

Using the real time open loop experimental data, the transfer function matrix is formulated as follows. The identified plant model from open loop (step test) input–output experimental data is

$$G_m(s) = \begin{pmatrix} \dfrac{1.0038e^{-3s}}{120s+1} & \dfrac{1.0536e^{-6s}}{279.5s+1} \\ \dfrac{1.0536e^{-6s}}{279.5s+1} & \dfrac{1.0038e^{-3s}}{120s+1} \end{pmatrix} \tag{29}$$

Similarly, relay feedback response model given by Eq. 5 can be simulated to generate theoretical response values and is then validated (Figure 6).

Parametric identification and model validation

For coupled tanks process, D_{21} and D_{22} are found to be 1 and 1, comparison of true value with estimated value is listed in Table 5 and validation is shown in Figure 6.

Validation of 3-by-3 MIMO process

The transfer function of multiproduct plant distillation column for the separation of binary mixture of ethanol–water (OR column) is given by

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \dfrac{0.66e^{-2.6s}}{6.7s+1} & \dfrac{-0.61e^{-3.5s}}{8.64s+1} & \dfrac{-0.0049e^{-s}}{9.06s+1} \\ \dfrac{-2.36e^{-3s}}{5s+1} & \dfrac{1.11e^{-6.5s}}{3.25s+1} & \dfrac{-0.01e^{-1.2s}}{7.09s+1} \\ \dfrac{-34.68e^{-9.2s}}{8.15s+1} & \dfrac{46.2e^{-9.4s}}{10.9s+1} & \dfrac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{30}$$

It is a 19 tray 12-inch diameter copper column having variable feed and side stream draw-off locations. It can be noted that $g_{p33}(s)$ component of the above transfer function can be approximated to a FOPDT model structure (using Skogestad[18] method) after which relay response can be generated using simulink (experimental).

Model validation and parametric identification

Similarly, relay feedback response model given by Eq. 9 can be simulated to generate theoretical response values and is then validated (Figure 7). It can be seen from the figure that the experimental (Simulink) and theoretically (relay response model) calculated responses match each other. Thereby, we validate the derived model. After validating the relay response analytical equation, next task is to identify model parameters of an unknown process. Let us consider that the OR column has been provided to us for identifying model parameters. Using relay signals, output response from closed-loop off-diagonal transfer function is recorded. From this output response, land mark points are detected and boundary conditions are formulated. These boundary conditions are used to frame 10 equations for identifying unknown process model parameters of 3-by-3 unknown system (OR Column). The estimated values of parameters are shown in Table 6. A process model is reconstructed using these estimated parametric values.

Sequential relay test is performed on this reconstructed model and off-diagonal closed-loop output response is collected. These output relay responses from reconstructed model and from original OR column are compared and is

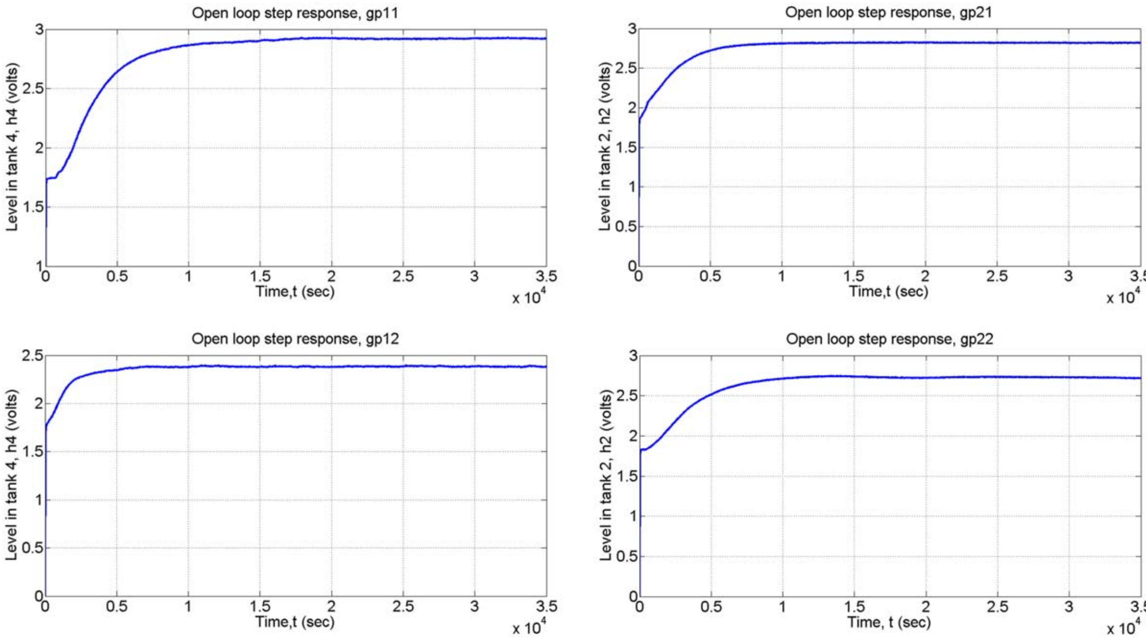

Figure 5. Change in levels (open-loop response) due to disturbances on input voltages (u_1 and u_2) of two pumps separately: process is liquid level control in coupled tanks system.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

shown in Figure 8. To find the efficiency of the estimation procedure developed here, a model error analysis need to be carried out as explained below.

Modeling error analysis on coupled tanks experimental setup

Mass balance equations for four individual tanks are constructed. These continuity equations are linearized around the operating point (steady-state liquid level). Thus, actual plant model for the real time coupled tanks system is derived from first principle method as

$$G_p(s)=\begin{pmatrix} \dfrac{0.0041}{(s+0.0017)(s+0.0023)} & \dfrac{0.0049}{(s+0.0023)(s+0.002)} \\ \dfrac{0.0041}{(s+0.0017)(s+0.0021)} & \dfrac{0.0041}{(s+0.002)(s+0.002)} \end{pmatrix} \tag{31}$$

The plant model from open-loop (step test conducted on individual tanks) input–output experimental data is identified as first-order plus dead time model structure system

$$G_m(s)=\begin{pmatrix} \dfrac{1.002e^{-3s}}{120.2s+1} & \dfrac{1.02e^{-6s}}{280.3s+1} \\ \dfrac{1.02e^{-6s}}{280.3s+1} & \dfrac{1.002e^{-3s}}{120.2s+1} \end{pmatrix} \tag{32}$$

In fact, liquid level system is identified as second-order dynamics, the above system has been approximated as first-order plus dead time models.

Multiplicative error between real data and model (Eq. 32 or Table 5) has been calculated. Multiplicative error is obtained using the formula given by Eq. 33

$$e_m=\int_{\omega_u/\sqrt{10}}^{\omega_u*10} \frac{\text{abs}\left(G_p(j\omega)-G_m(j\omega)\right)}{G_p(j\omega)}d\omega \tag{33}$$

For the liquid level coupled tanks system, multiplicative error is found to be as

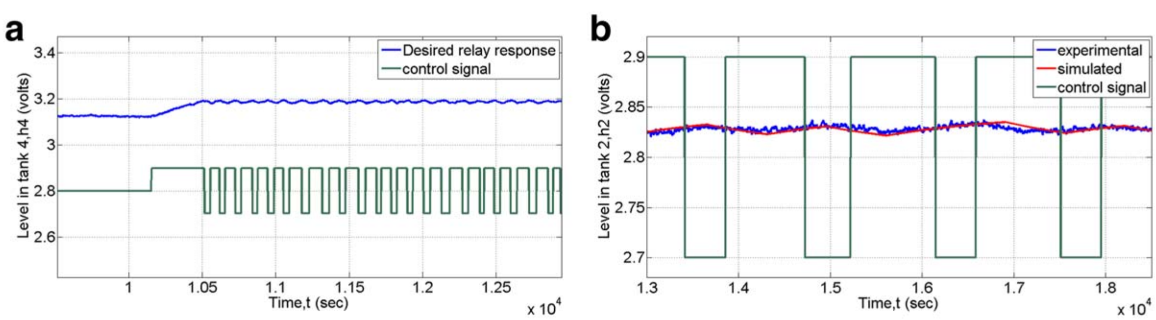

Figure 6. (a) Desired experimental relay response of coupled tanks experimental setup. (b) Validation of analytical expression (dashed) of 2-by-2 interactive cross transfer function (TF) with real time experimental off-diagonal closed-loop relay response (solid) in an enlarged view.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Table 5. Comparison of True and Estimated Values of Coupled Tanks System

Coupled Tanks Process	True Value	Estimated Value
k_{21}	1.0536	1.02
t_{21}	279.5	280.3
k_{22}	1.0038	1.002
t_{22}	120	120.2

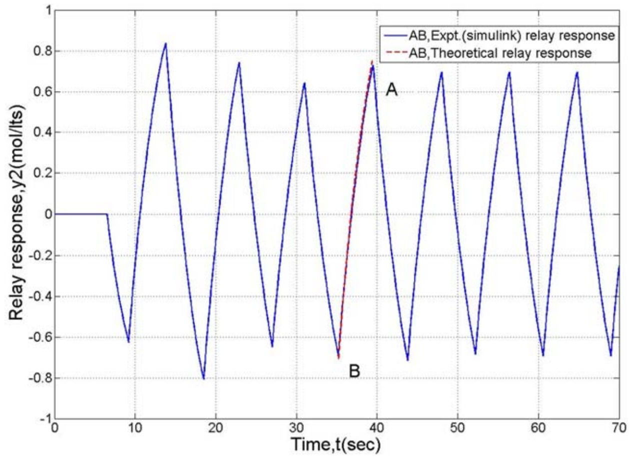

Figure 7. Validation of analytical expression (dashed) of 3 × 3 interactive cross transfer function with experimental (simulated) biased relay response (solid) for OR column.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

$$e_m=\begin{pmatrix} 3.0768e-006 & 3.4474e-005 \\ 3.4474e-005 & 3.0768e-006 \end{pmatrix} \tag{34}$$

From this result, it is evident that the identified model is in good agreement with the real time coupled tanks system.

Comparison of the present method with available methods

Koo et al.[19] reported results on identification of process model parameters of WB column. Their identified model is compared with the identified (WB Column) model by present method. Multiplicative errors in both the cases are evaluated around ω_u value ($\omega_u = 1.5714$), and it is found that the present method yields less error compared to Koo et al.[19] The estimated values of WB-model parameters are provided in Table 7.

Table 6. True and Estimated Process Parameters of OR Process

S. No.	Parameters	True value	Estimated value
1	k_{21}	1.11	1.05
2	t_{21}	3.25	3.2
3	D_{21}	6.6	6.6
4	k_{22}	−2.36	−2.11
5	t_{22}	5	5.003
6	D_{22}	3	3
7	k_{23}	−0.01	−0.01
8	t_{23}	7.09	7.134
9	D_{23}	1.2	1.2
10	k_{31}	−34.68	−34.68
11	t_{31}	8.15	8.09
12	D_{31}	9.2	9.2
13	k_{33}	0.68121	0.69
14	t_{33}	5.06	4.99
15	D_{33}	1	1

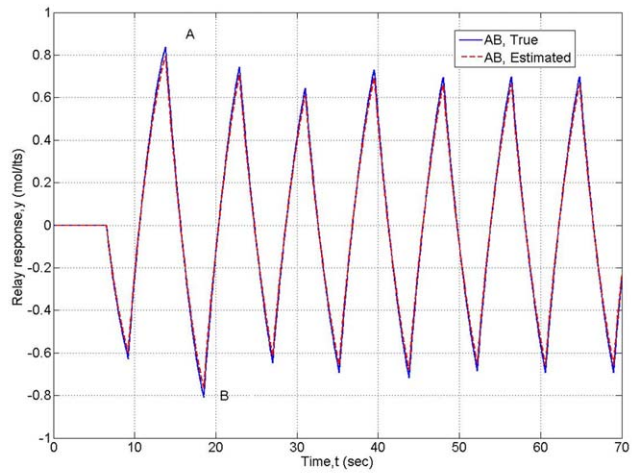

Figure 8. Validation of closed-loop off-diagonal relay responses obtained from identified 3 × 3 model (dashed line: B) with that obtained from actual OR process (solid line: A).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Effect of noise on MIMO process

To determine efficacy of the present identification procedure, it is necessary to test the method with output collected under measurement noise. In a realistic environment, the major concerns for any identification method are measurement noise. Here, the measured process output is corrupted by normally distributed random noise of 1% (variance of

Table 7. Comparison of True and Estimated Values of WB Column

WB Process	*True Value*	*Estimated Parameters*	
		Proposed Method	*Koo et al. Method*[19]
k_{21}	6.6	6.58	6.679
t_{21}	10.9	11	11.042
D_{21}	7	7	6.886
k_{22}	−19.4	−19.408	−19.522
t_{22}	14.4	14.388	14.637
D_{22}	3	3	2.970
Multiplicative Error	0	$\begin{pmatrix} 2.4796e-006 & 9.9008e-005 \\ 6.0249e-005 & 6.2210e-006 \end{pmatrix} = 5.9497e-009$	$\begin{pmatrix} 4.8751e-004 & 0.0069 \\ 0.0014 & 3.7446e-004 \end{pmatrix} = 9.4774e-006$

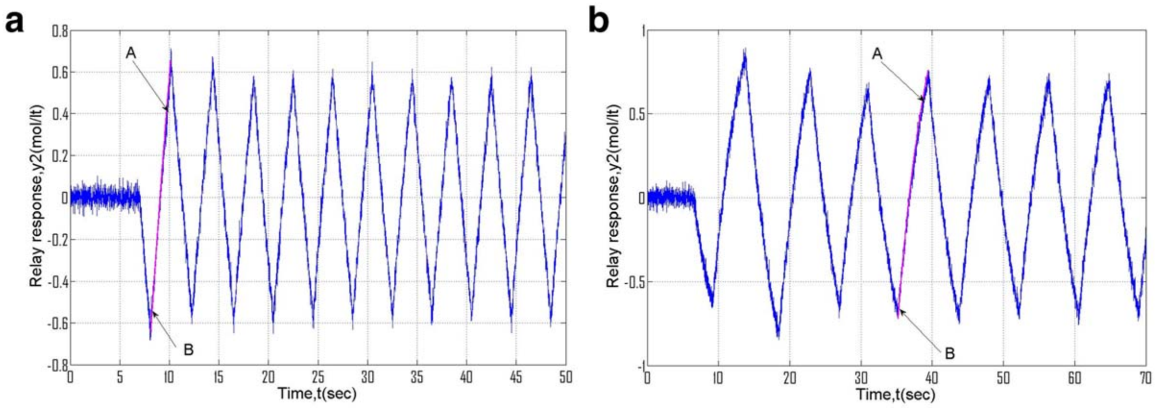

Figure 9. (a) Validation of analytical expression of 2 × 2 interactive cross TF with experimental relay response in the presence of measurement noise (WB column); (b) validation of analytical expression of 3 × 3 interactive cross TF with experimental relay response in the presence of measurement noise (OR column).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

0.001). The validated off-diagonal closed-loop relay response in the presence of measurement noise for 2-by-2 and 3-by-3 MIMO systems is shown in Figures 9a, b, respectively.

Conclusions

Analytical expressions for relay feedback responses are obtained by assuming that periodic relay responses are due to small step changes in input to the process transfer functions. Based on the roots of denominator of interactive (closed-loop off-diagonal) process transfer function, $g_{p12,cl}$, theoretical model equations are used for computation of oscillatory relay responses of 2-by-2 and 3-by-3 MIMO systems. An example has been considered in rationalizing the developed model-equations. When the relay input and outputs are synchronized, the response curves help us to find out sufficient land mark points that provide us sufficient information for model parameter identification. These models are validated against off-diagonal closed-loop relay response from liquid level coupled tank system. Using model-equations, land-mark points, area of relay response, moment method, and some curve properties, parameter estimation algorithms have been formulated for these multivariable systems. Thus, the bias value has been added to model response in order to match with the experimental response, and the parameters are estimated based on the shifted model response. Also, the achievable accuracy is justified by multiplicative error that is computed between real data and model. These results are provided in a closed form solution for the first time.

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